Errata in *The Theory of Probability*

Sadly, errors are apparently inescapable in a first edition of a volume of this size and scope. My students and generous readers have helped me track down some of them: here is a current list of errors and ambiguities that have been discovered identified by page and line number (positive counting from the top of the page, negative from the bottom of the page) and organised by chapter.

A round two dozen or so algebraic errors have been discovered so far accompanied by a few that are purely cosmetic; the bulk of these errors, fortunately, appear to be minor and unlikely to seriously perturb a student as the corrections are obvious in context. There are no doubt more errors lying in wait to trip up the unwary reader and I can only apologise in advance for the befuddlement and consternation these may cause. I would very much appreciate receiving word of new errors and ambiguities as they are discovered. Without further ado here are the corrections.

I: Probability Spaces

p. 15, l. -16 The line before the definition should conclude in "Summe". The capitalisation of the German noun was pointed out to me by Sonia Soro.

II: Conditional Probabilities

p. 52, l. -2 The footnote citation should be "Über ... Einwände ...". Courtesy Sonia Soro.

III: A First Look at Independence

p. 86, l. 22 The product space referred to at the start of this line should read "... space $\Omega = \mathfrak{A}^{\binom{n}{2}}$ as the $\binom{n}{2}$-fold product ..." (instead of $\Omega = \mathfrak{A}^n$). Pointed out by Amin Rahimian.

IV: Probability Sieves

p. 109, l. -15 The parenthetical comment at the start of the line should read "(Our problem had $n = 7$ and $k = 3$.)" Instead of $k = 2$. Courtesy Amin Rahimian.

p. 120, l. 10 The probability that the vertex i is isolated should be given by $\mathbf{P}(A_i) = (1-p)^{n-1}$ (and not $2^{-(n-1)}$ as written). Again, courtesy Amin Rahimian.

p. 124, l. 12 A graph was misidentified in the sentence begun in the middle of this line: it should read "By merging G_1 and G we now form a larger graph G_2 ...". Once more, courtesy Amin Rahimian.

V: Numbers Play a Game of Chance

p. 162, l. 11 There is a missing binomial coefficient on the right of the displayed equation in Problem 16:

$$\int_0^1 f\Big(\frac{z_1(t;x)+\cdots+z_n(t;x)}{n}\Big)\,dt = \sum_{k=0}^{n} f\Big(\frac{k}{n}\Big)\binom{n}{k}x^k(1-x)^{n-k}.$$

VI: The Normal Law

p. 189, l. 14 The display equation immediately following the Local Limit Theorem should read: "The local limit theorem provides an alternative path to de Moivre's theorem. It is clear that for $|k| \le K_n$,

$$\phi(x) \sim \phi\left(\frac{k}{\frac{1}{2}\sqrt{n}}\right)$$

uniformly for all x in the interval ...".

VII: Probabilities on the Real Line

p. 210, l. 8 Add the clarification: "*... where the map ζ is monotone and differentiable (that is to say, it is differentiable and either increasing or decreasing) over the support of* X."

VIII: The Bernoulli Schema

p. 244, l. 1 Replace"is at least $n \ge 1/4(p-q)^2\delta^2$ which is in excess of 10^{10}" by "is at least $n \ge 1/4(p-q)^2\delta$ which is in excess of 2×10^9".

p. 252, l. 4 Replace $2tn$ by $2t\nu$. Pointed out by Min Wen.

IX: The Essence of Randomness

p. 288, l. 5 Change the sign of the cosine in penultimate paragraph of Section IX.3: "... length of the received vector is $2-2\cos\Theta$".

p. 312, l. -3 Replace the words "complex roots" by "has a pair of complex conjugate roots" to avoid confusion. Alternatively, we could replace "complex roots" by "no real roots". [A quadratic always has two complex roots. We want to eliminate the possibility that the roots are real.]

p. 312, l. -8 The last line of Problem 19 should be "Determine the density of $X-T$". Courtesy Shuotian Cheng.

p. 312, l. -5 To be notationally consistent move the positive part symbol to the subscript: write $W = (X-T)_+$.

p. 312, l. -1 The last line should conclude with "Show that $(U_1, \ldots, U_{n-1})$ has the same distribution ...".

X: The Coda of the Normal

p. 334, l. -10 There is a small correction to the displayed equation (5.2) which should read

$$T = \frac{\hat{M} - m}{\hat{\Sigma}/\sqrt{n}} = \frac{\hat{M} - m}{\sigma/\sqrt{n}} \Bigg/ \frac{\sqrt{(n-1)\hat{\Sigma}^2/\sigma^2}}{\sqrt{n-1}} \tag{5.2}$$

(In the third term, the expression $\sigma\sqrt{n}$ in the original equation should be $\sigma/\sqrt{n}$ as shown here.)

XII: Random Variables

p. 408, l. 9–11 The limiting operations lim inf and lim sup are interchanged in the paragraph above Theorem 2. The equation in line 9 should read $\liminf_n A_n = \lim_n B_n = \bigcup_n B_n$; the equation in lines 10–11 concluding the paragraph should read $\limsup_n A_n = \lim_n C_n = \bigcap_n C_n$. Courtesy Vasileos Tzoumas.

p. 425, l.-5 In Problem XII.16, the condition to be verified should be: "Show that $\mathbf{P}\{L \le t\} = t^2/4$ for $0 < t < 2$".

p. 426, l. 8 In Problem XII.19, the expression for the empirical distribution should be written in the form "$F_n(x) = \frac{1}{n}\sum_{j=1}^{n} 1(X_j \le x)$". [The normalising fraction $1/n$ is missing in front of the sum in the text.]

XIII: Great Expectations

p. 451, l. 15 Strengthen the modulus inequality to read "$\big|\mathbf{E}(X)\big| \le \mathbf{E}\big(|X|\big)$".

p. 459, l. 3 Replace division by log 2 by multiplication by log 2 in the numerical estimate of Quicksort's overhead in the middle of the line. The line should read "... factor of only $2\log 2 \approx 1.386$ over the best result ...".

XIV: Variations on a Theme of Integration

p. 465, l. -22 The Latin title of the section should read UTILE ERIT SCRIBI $\int$ PRO OMNIA. Enrico Lumetti pointed out to me that scribi is the passive infinitive while scribit would have been the third person singular in the future tense. The full phrase which provides the context is "Utile erit scribi $\int$ pro omnia, ut $\int l = omn.l$, id est summa ipsorum l." It appears in an unpublished manuscript of Leibniz.

p. 520, l. 16, l. -12 In line 16 the hint for Problem 36 should read "Replace $Y = |X|^s \ldots$". There is a missing comma in line -12 in Problem 41: write $\mathrm{Cov}\big(f(\mathbf{X}), g(\mathbf{X})\big)$ instead of $\mathrm{Cov}\big(f(\mathbf{X})g(\mathbf{X})\big)$ [courtesy Shih-ling Phuong].

p. 522, l. -18 The definition of a martingale above Problem 54 should read "$\ldots$ X_n is integrable, measurable with respect to $\mathcal{F}_n \ldots$".

XV: Laplace Transforms

p. 525, l.-14 In the expression for the negative binomial distribution, the upper index in the binomial coefficient should be $-r$ instead of r: the expression for the distribution should read $\binom{-r}{k}(-q)^k p^r$. Courtesy Amin Rahimian.

p. 526, l. -14, l. -12 The parameter h should be replaced by $|h|$ at a couple of places in the proof in small print. The inequality in line -14 should read "$|(e^{-hx} - 1 + hx)/h| \le |h|x^2/2$". The expression at the right end of the display equation on line -12 should read

$$\le \int_{[0,\infty)} x^n e^{-\zeta x} \left| \frac{e^{-hx} - 1 + hx}{h} \right| dF(x) \le \frac{|h|}{2} \int_{[0,\infty)} x^{n+2} e^{-\zeta x}\, dF(x).$$

p. 558, l. -13 Interchange the rôles of p and q in the second line of the displayed equation in Problem 10:

$$\mathfrak{G}_n(s) = \begin{cases} \frac{n-(n-1)s}{n+1-ns} & \text{if } p = q = 1/2, \\ \frac{p\left(q^n - p^n - qs(q^{n-1} - p^{n-1})\right)}{q^{n+1} - p^{n+1} - qs(q^n - p^n)} & \text{if } p \ne q. \end{cases}$$

p. 560, l. 19 Replace the first line of Problem 27 by "If $p = 1/2$ show that $\mathfrak{F}_r(s) = s \cdot \mathfrak{F}_{r-1}^{(1)}(s)$ and $\ldots$".

p. 560, l. -1 The equation concluding the last line of Problem 31 should read "$\widehat{F}^{(n)}(0) = i^n\, \mathbf{E}(X^n)$".

XVI: The Law of Large Numbers

p. 586, l. 15 Replace a_n by α_n in the first displayed equation on this page:

$$|X_1 + \cdots + X_n - \alpha_n| \le |U_1 + \cdots + U_n - \alpha_n| + |V_1 + \cdots + V_n|.$$

p. 601, l. 9 Minor typographical error: the line should read "unboundedly large samples then $V_{\mathfrak{A}}$ *is* the largest integer $\ldots$" not 'as the largest integer'. Courtesy Amin Rahimian.

p. 604, l. -13, l. -12 In Problem 4, line -12, remove the word "large". (It suffices to say "Suppose a is a positive constant $\ldots$".) In line -13, replace "Var(J)" by "$\mathrm{Var}\big(g(X)/f(X)\big)$".

XVII: From Inequalities to Concentration

p. 625, l. -4 Replace "$\rho_0(x, \mathbb{A}) = 1_{\mathbb{A}}(x)$ is the indicator for the set $\mathbb{A}$" by "$\rho_0(x, \mathbb{A}) = 1_{\mathbb{A}^c}(x) = 1 - 1_{\mathbb{A}}(x)$ is the indicator for the complement of the set $\mathbb{A}$". Pointed out by Amin Rahimian.

p. 646, l. 11 In Problem 1, improve the conclusion by a factor of two to read $|\mu - M| \le \sqrt{\pi}/\big(2\sqrt{\alpha}\big)$.

XVIII: Poisson Approximation

p. 659, l. 11 In Theorem 2, the inequality given should be $\Delta g \le \lambda^{-1}(1 - e^{-\lambda}) \le \min\{\lambda^{-1}, 1\}$ as is made clear in the proof. The term in the middle inequality is incorrectly stated as $\lambda^{-1}(1 - e^{-1})$. Courtesy Amin Rahimian.

p. 665, l. 5 The end of the line should read $\Gamma'_\alpha = \Gamma \setminus (\Gamma_\alpha \cup \{\alpha\})$ and not $\Gamma \setminus (\Gamma'_\alpha \cup \{\alpha\})$. Due to Amin Rahimian.

p. 665, l. -11 As before, the set Γ'_α should be defined by $\Gamma'_\alpha = \Gamma \setminus (\Gamma_\alpha \cup \{\alpha\})$ (and not $\Gamma_\alpha \setminus (\Gamma_\alpha \cup \{\alpha\})$ as written). Again, due to Amin Rahimian.

p. 666, l. 8 Theorem 1 is about triangles (with $k = 3$) but the statement anticipates Theorem 2 on page 668 dealing with cliques of size k. In the second line of Theorem 1, λ should be defined by $\lambda = \binom{n}{3}p^3$ (replace k by 3 in the statement). Once more, due to Amin Rahimian.

XIX: Convergence in Law, Selection Theorems

p. 715, l. -2, -3 The ending page number for the Hardy and Wright reference is incorrectly stated. The page reference should be "pp. 375–393".

p. 716, l. –8, -9 A rewording will eliminate a slight ambiguity in language: "(If ϑ is rational and equal to a/b, say, a regular polygon with vertices at $0, 1/b, \ldots, (b-1)/b$ will serve.)"

p. 716, l. -3 One (obvious) correction replacing F by $F^{\star k}$ and one small clarification. Replace the line by "Suppose the jump of $F^{\star k}$ at x' has size p. Then $0 < p < 1$ with p bounded away from both one and zero because there are at least two points of jump."

XX: Normal Approximation

p. 720, l. -12 There is a multiplicative factor $n^{-1/2}$ missing in the final expression for S_n^*. The equation should say $S_n^* = (S_n - n\mu)/\sqrt{n}\,\sigma = n^{-1/2}\sum_{k=1}^{n}(X_k - \mu)/\sigma$. Pointed out by Amin Rahimian.

XXI: Sequences, Functions, Spaces

p. 786, l. -7 A second vertical bar to represent a norm is missing in the middle of the line: the line should read "... and g, respectively, that is, $\|f - f_N\| \to 0$ and ...". Courtesy Amin Rahimian.