Problems for Chapter 8 of 'Ultra Low Power Bioelectronics'

Problem 8.1

A first-order low-pass filter with a corner frequency of 100 kHz has an input-referred voltage-noise power spectral density of $\overline{v_{in}^2(f)} = 36\,\text{nV}/\sqrt{\text{Hz}}$. Calculate the input-referred rms noise voltage of this filter. Repeat the calculation if the noise is not flat but exhibits flicker-noise behavior with a $1/f$ -noise corner frequency of 10 kHz (you can assume a start frequency for integration of 1 Hz). What percentage of the input-referred rms noise comes from flicker noise in this case?

Problem 8.2

Consider the common-source amplifier circuit in Figure P8.2.

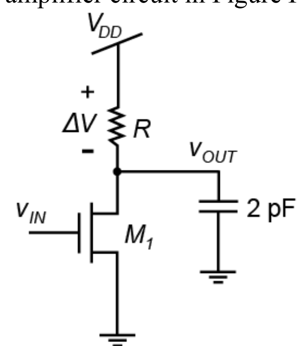

Figure P8.2: A common-source amplifier.

a) Assume that transistor M_1 is sized and properly biased in weak inversion, and that V_{DD} is maintained at a large enough value such that M_1 is operating in saturation. Also assume that M_1 has infinite output resistance. Find the DC-voltage gain from v_{in} to v_{out} in terms of κ, ΔV, and ϕ_t.
b) Calculate the short-circuit current noise power spectral density (PSD) at the v_{OUT} node due to resistor R. Also calculate the short-circuit current noise PSD at v_{OUT} due to transistor M_1.
c) Determine the input-referred noise PSD at the v_{IN} node. At what value of ΔV does the noise contribution from M_1 become exactly equal to the noise contribution from the resistor? Determine the voltage gain at which such equality is achieved.
d) Suppose we design this circuit to have a voltage gain of 5, while biasing M_1 in the weak-inversion regime with a drain current of 1 μA. Determine the input-referred rms noise of this circuit. What percentage of this noise does the resistor contribute? What is the minimum detectable signal at the input node in the passband?

Problem 8.3

Consider the cascode transistor in the Figure P8.3. The circuit is normally used as a high-impedance current source in analog circuits. In this problem, we will analyze the effect of adding a cascode transistor to the output noise of the circuit.

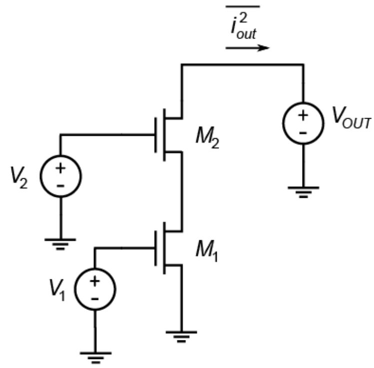

Figure P8.3: A cascoded common-source amplifier.

a) Denote the noise generator in M_1 and M_2 as $\overline{i_{n1}^2(f)}$ and $\overline{i_{n2}^2(f)}$ respectively. Assume that V_{OUT} and V_2 are kept high enough such that all transistors are operating in saturation. Draw a small-signal equivalent of the circuit to include $\overline{i_{n1}^2(f)}$ and $\overline{i_{n2}^2(f)}$ as inputs and $\overline{i_{out}^2(f)}$, the short-circuit output current noise PSD at the drain of M_2, as output. Do not neglect transistors' output impedances and remember to include the g_{mb} generator for M_2.

b) Find and expression for $\overline{i_{out}^2(f)}$ in terms of $\overline{i_{n1}^2(f)}$, $\overline{i_{n2}^2(f)}$ and the circuit's small-signal parameters. From your answer, how does the addition of M_2 affect the output noise current PSD of the circuit? Under what conditions can we neglect the noise contribution from M_2?

c) Suppose V_1 and the W/L of M_1 are chosen such that M_1 is saturated and operates in weak inversion with a drain current of 100 nA . Calculate the output current noise PSD $\overline{i_{out}^2(f)}$. You can assume that $g_m r_o >> 1$ for all transistors in the circuit. Explain qualitatively how $\overline{i_{out}^2(f)}$ is affected if V_2 is lowered such that M_1 goes into its linear region, while M_2 is still in saturation.

Problem 8.4

Consider the circuit shown in Figure P8.4. This circuit is used to compute a weighted-average of its input voltages. The transconductance amplifiers (OTA) G_1 and G_2 are implemented with the standard five-transistor topology shown in Figure 8.8 (b). Assume that all transistors are operating in subthreshold, $\kappa_n = \kappa_p = 0.7$, and that $\phi_t = 26$ mV.

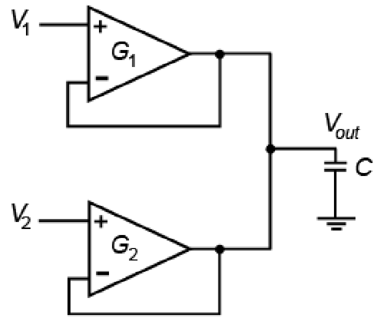

Figure P8.4: An averaging circuit.

a) Find the transfer function from $V_1(s)$ and $V_2(s)$ to $V_{out}(s)$ in terms of G_1, G_2, and C.
b) If G_1 and G_2 are biased such that their total current are I_{B1} = 200 nA and I_{B2} = 50 nA respectively, determine the corner frequency of the circuit. What is the corresponding noise bandwidth?
c) Assume the bias settings in part b). Compute the input-referred voltage-noise power spectral density, $\overline{v_{n1}^2(f)}$ and $\overline{v_{n2}^2(f)}$, for G_1 and G_2.
d) Assume that the noise sources in G_1 and G_2 are uncorrelated. If C= 10 pF, find a numerical value for the rms voltage noise at the V_{out} node. What is the minimum detectable signal at the V_1 node in the passband if V_2 is tied to a noiseless dc voltage source?

Problem 8.5

Let's consider the super-buffer circuit in the Figure P8.5. This circuit uses a high-gain feedback loop to significantly reduce impedance at the V_{out} node (the output impedance). The circuit is configured such that $I_2 > I_1$ and a current of $I_2 - I_1$ flows through M_2 . Assume that I_1 and I_2 have infinite output impedances throughout this problem.

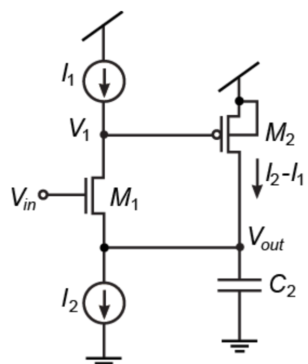

Figure P8.5: A super-buffer circuit.

a) Draw a small-signal equivalent circuit and feedback block diagram with V_{in} as the input and V_{out} as the output. Remember to include transistors' output impedances and the body effect of M_1 (include its g_{mb} generator). You may assume that C_2 is the only significant capacitance.

b) From the block diagram in a), calculate the output impedance $Z_{out}(s)$ by applying an appropriate test voltage/current source at the node V_{out}.
c) Suppose I_1 has an output current noise spectral density of $2qI_1$. We can model it as having a noise generator with a power spectral density of $2qI_1$ in parallel with the current source I_1. Show that the magnitude of the transfer function from this noise generator to the short-circuit output current is $|1 - g_{m2}r_{o1}|$. (HINT: Put a dc voltage source at V_{out} to make it ac ground, then calculate the small-signal current flowing into the dc source).
d) Add M_1's noise generator. Show that the magnitude of the transfer function of this noise generator to the short-circuit output current is $|g_{m2}r_{o1}|$.
e) Finally, add M_2's noise generator and a $2qI_2$ noise generator in parallel with I_2. Compute the total noise power spectral density $\overline{v_{out}^2}$ at the V_{out} node in terms of circuit parameters and $Z_{out}(s)$.
f) Find the rms voltage noise at the V_{out} node. What is the minimum detectable signal at V_{in} in the passband?

Problem 8.6

Prove that the effective noise bandwidth for a second-order low-pass filter described by $H(s) = \dfrac{1}{(1+\tau s)^2}$ is $\dfrac{1}{8\tau}$ Hz.

Problem 8.7

In this chapter, we have seen how the inclusion of the band-pass amplifier in Figure 8.16 helps alleviate the effect of parasitic capacitance at the output node of a MEMS capacitive sensor. In this problem, we will study how this parasitic capacitance degrades the noise performance of the bandpass amplifier. Figure P8.7 shows a simplified schematic of a capacitively-coupled bandpass amplifier. The topology shown in the figure is a very popular one for neural-recording amplifier design. Assume that there is a parasitic capacitance C_p from the V_- terminal to ground. To ease the analysis, assume that $C_{out} >> C_2$.

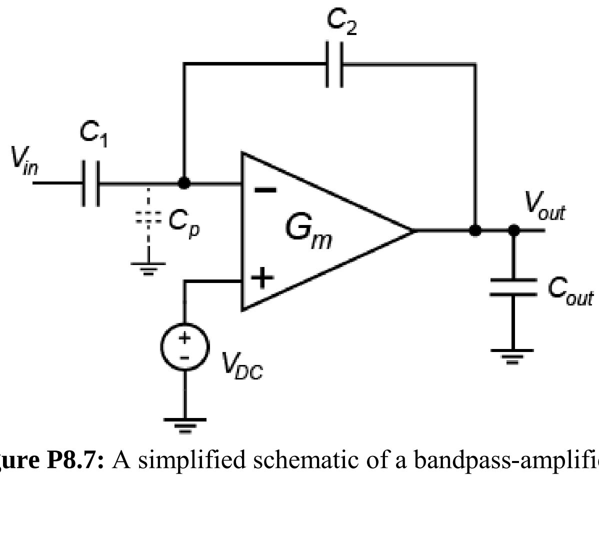

Figure P8.7: A simplified schematic of a bandpass-amplifier.

a) Using superposition, by grounding V_{out}, find the transfer function from V_{in} to V_-. Similarly, by grounding V_{in}, find the transfer function from V_{out} to V_-.
b) Based on the answer in part a), draw a small-signal feedback block diagram and find the transfer function from V_{in} to V_{out}.
c) Suppose the input-referred noise PSD of the op-amp is given by $\overline{v_n^2(f)}$. Add this noise source in your feedback block diagram. Calculate the input-referred noise power spectral density of the overall amplifier, $\overline{v_{n,in}^2(f)}$. How does the parasitic capacitance C_p affect the input-referred noise of the overall amplifier?
d) In the fabrication process that you are using only a poly-poly capacitor with a parasitic bottom-plate capacitance to ground, which is 15% of the total capacitance (intended plus parasitic), is available. Assume that the op-amp is designed to have an input-referred noise PSD of $40\ \text{nV}/\sqrt{\text{Hz}}$. Neglecting the input capacitance of the op-amp, calculate the rms input-referred noise of the overall amplifier if the bottom plates of C_1 and C_2 are connected to V_-. How would the result change if the top-plate of C_1 and C_2 instead are connected to V_-?

<u>Problem 8.8</u>
In this problem, we will study a low-noise design technique for an operational transconductance amplifier. We will compare the OTA in Figure P8.8 below and the basic one in Figure 8.8 (b) in terms of their noise performance for a given total bias current. Assume that all transistors are operating in weak inversion and in the saturation region. You can assume that each transistor has $g_m r_o >> 1$.

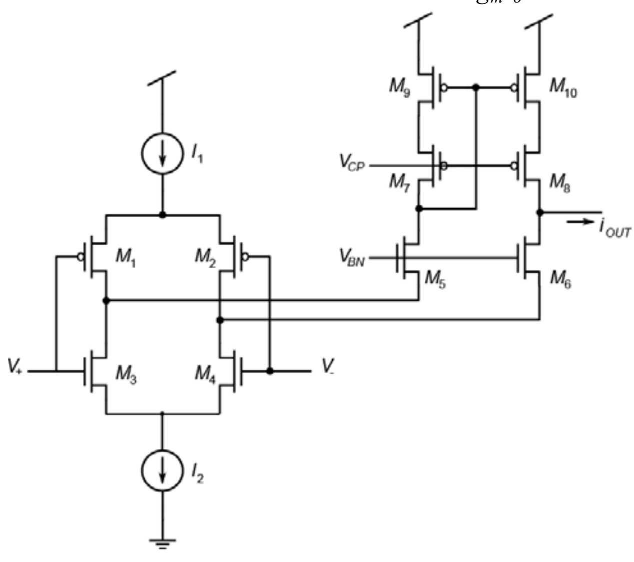

Figure P8.8: Low-noise OTA.

a) In this figure, compute the total short-circuit output current noise PSD in terms of I_1, I_2, and small-signal transistor parameters. What is the effective G_m of the OTA? Explain qualitatively how the effective G_m would change if $I_2 - I_1$ is very small and the $g_m r_o$ of the transistor is not much greater than 1. (HINT:

Consider current division at the drains of the input M_1 and M_3 transistors and the source of M_5 and also for corresponding transistors in the other differential arm).

b) Compute the input-referred voltage noise PSD of the OTA .

c) Suppose $I_2 = 1.2\ \mu A$ and $I_1 = 1\ \mu A$. Calculate numerically the input-referred voltage noise PSD of this OTA. For the same total current, compare your result with that of the OTA in Figure 8.8 (b). Is it surprising that a circuit with many more transistors can have lower input-referred noise for the same power?

Problem 8.9

Use the equipartition theorem to compute

a) The total rms thermal voltage noise seen across the capacitor of a parallel LCR resonator. Express your answer in terms of kT, R, L, and C.

b) The total rms thermal current noise seen flowing through the inductor in a parallel LCR resonator. Express your answer in terms of kT, R, L, and C.

c) The total thermal rms velocity noise observed in a spring-mass-damper mechanical system with mass m, spring stiffness k, and mechanical damping η.

d) The total thermal rms displacement noise observed in a spring-mass-damper mechanical system with mass m, spring stiffness k, and mechanical damping η.

e) By mapping mechanical spring compliance ($1/k$) to electrical inductance L, mechanical mass to electrical capacitance C, and mechanical damping to electrical conductance $G=1/R$ in a parallel LCR resonator, show that your answers to parts c) and d) are consistent with those of parts a) and b) respectively.

Problem 8.10

This problem requires the use of a circuit simulator such as SPICE. Design a first-order low-pass filter similar to the one shown in Figure 8.12, using the OTA topology of Figure 8.8. Bias the OTA such that the effective G_m is 100 nA/V and use a load capacitance of $C = 10$ pF. Calculate the total rms output voltage noise. Perform a SPICE simulation to verify your hand calculation.