Problems for Chapter 3 of 'Ultra Low Power Bioelectronics'

Problem 3.1

Simplify Figure 3.4 in the different regions of operation of a long-channel MOS transistor: For each case below, draw only the dominant drift/diffusion current source, state whether the capacitors are fixed or vary with surface potential, and state whether the source-to-channel junction and drain-to-channel junction may be approximated with a fixed bias across them or not:

a) Sub-threshold
b) Linear (above threshold)
c) Saturation (above threshold)

Problem 3.2

This problem will study the built-in potential of a pn junction. The built-in potential is defined as the potential difference from the n-side to the p-side in a pn junction caused by the depletion region in the junction. Assume that the intrinsic carrier concentration for silicon, n_i, is 10^{10} cm^{-3}.

a) What is the concentration of donors if the built-in potential is 720 mV, the concentration of acceptors in the p-side equals 10^{18} cm^{-3}, and we assume that the temperature is 300 K?
b) How much does the built-in potential increase if the concentration of donors found in part a) increases by a factor of 10?

Problem 3.3

This problem will study how the electric field, potential, and charge density vary for the MOSCAP shown in Figure 3.10, assuming that the charge-sheet approximation is valid, that the surface potential of the MOSCAP is at ψ_S, and that the MOSCAP is in strong inversion.

a) Plot the depletion charge density as y varies from 0, right at the channel surface, to y_D, the edge of the MOSCAP depletion region.
b) Using Gauss's law, plot the electric field $E(y)$ as y varies from 0 to y_D.
c) Finally, plot the potential $\Psi(y)$ as y varies from 0 to y_D.

Problem 3.4

Assume that we have a transistor with $W = L = 15\ \mu$m in a 0.5 μm process. Assume that the drain and source are grounded, and that $T = 300$ K, $n_i = 10^{10}$cm^{-3}, ε_{ox}=3.9, and ε_{si}=11.7.

a) Calculate the surface potential in strong inversion, given $N_A = 4\text{x}10^{16}$ cm^{-3}.
b) Calculate γ, given $t_{ox} = 1.4 \times 10^{-8}$ m.
c) Calculate C_{dep}.

Problem 3.5

This problem will study how the depletion depth varies in a MOSCAP as its gate voltage varies.

a) By equating the total depletion charge in the bulk of Figure 3.10 to the depletion charge given by Equations (3.10) and (3.11), compute the depletion depth of the bulk charge as a function of the surface potential.
b) Plot the MOSCAP depletion width as the gate voltage is increased from

0 V to the maximum allowed voltage before oxide breakdown. Your plot does not need to be quantitatively accurate but should have the right qualitative shape based on your answers from part a) and the graphs of Figure 3.11.

Problem 3.6

This problem requires the use of a circuit simulator such as SPICE. This problem will study the differences between using an n-channel MOSFET as a capacitor versus using a regular capacitor in layout.

a) Create a 1 pF capacitor using an n-channel MOSFET with its drain and source connected to ground, and the gate connected to 5V. Note the area required in the simulator.
b) Create a (polysilicon-to-polysilicon) 1 pF capacitor. Note the area required in the simulator.
c) Compare the area required for creating the capacitor in part a) versus that in part b).

Problem 3.7

In this problem we will study how we can simplify Equation (3.23) for values of surface potential in Figures 3.11(a), (b), (c), or (d) that correspond to the central portion of the curves in these figures. In this region, since the variation in charge density with surface potential is gentle we will attempt to approximate it with a straight line.

a) Using Taylor-series approximations, linearize $\psi_s + \gamma\sqrt{\psi_s}$ at the operating point ψ_{se} so that you can rewrite this expression as $A*\psi_s + V_O$. Specify values for A and V_O.
b) Using Equation (3.21) and the linearized term for surface potential, re-derive a simpler form of Equation (3.23).
c) Comment on the differences between the equation you derived in part b) and Equation (3.23).

Problem 3.8

This problem explores why negative feedback results in a nearly constant 'diode-clamped' surface potential that is relatively invariant with charge density:

a) Assume that we are operating in very strong inversion and that Equation (3.12) may be approximated as $Q_I = -\gamma C_{ox}\sqrt{\phi_t e^{(\psi_s - 2\phi_F)/\phi_t}}$. Find an expression for the surface potential ψ_S as a function of the inversion charge Q_I.
b) Interpret how your derived equation is in accord with Figures 3.11 (c) and (d), and why it is consistent with strong negative feedback for values of ψ_S greater than $2\phi_F$.

Problem 3.9

Assuming that only one of the drift or diffusion generators is dominant in strong-inversion or weak-inversion respectively, simplify Equation (3.23) for each region of operation by ignoring the current-source term that does not dominate.

Problem 3.10

In this problem, we shall use feedback to quantitatively explore how $\Delta\psi_s$ varies with Δv_G in both weak inversion and in strong inversion.

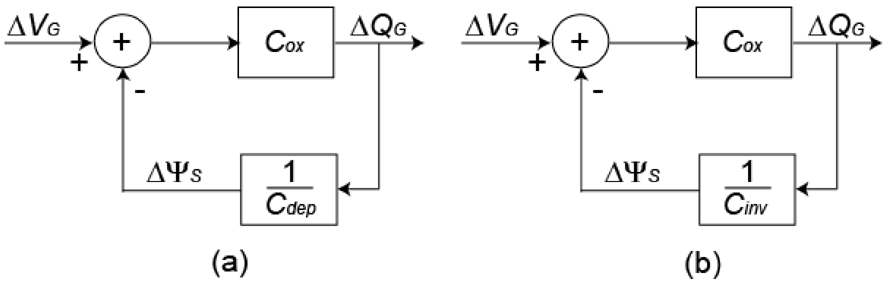

Figure P3.10 Feedback block diagram in a MOSCAP for (a) weak inversion; and (b) strong inversion.

a) Figure P3.10 (a) shows the feedback loop that relates Δv_G, $\Delta\psi_s$, and ΔQ_G in weak inversion, obtained from the capacitive divider of Figure 3.12. The variables all represent small deviations around an equilibrium operating point. Compute the relationship between $\Delta\psi_s$ and Δv_G using Black's formula.

b) Figure P3.10 (b) shows the feedback loop that relates Δv_G, $\Delta\psi_s$, and ΔQ_G in strong inversion. Assuming that only the exponential term is significant on the right hand side of Equation (3.12) (as in Problem P3.8), derive that $C_{inv} = -dQ_I / d\psi_S = |Q_I| / 2\phi_t$, where C_{inv} represents the effective capacitance in strong inversion present in the channel. Compute the relationship between $\Delta\psi_s$ and Δv_G using Black's formula. Explain why the 'diode-clamping' strong-inversion capacitance C_{inv} keeps $\Delta\psi_s$ relatively invariant with Δv_G in strong inversion.

c) Explain why C_{dep} is the slope of the monotonically increasing curve in Figure 3.11 (b) where the straight line intersects it. Explain why $-C_{ox}$ is the slope of the straight line at this same point. From geometric considerations, explain why $\Delta\psi_s$ and Δv_G at this intersection point are related as predicted by the weak-inversion result of part a).

d) Explain why C_{inv} is the slope of the monotonically increasing curve in Figure 3.11 (d) where the straight line intersects it. From geometric considerations, explain why $\Delta\psi_s$ and Δv_G at this intersection point are related as predicted by the strong-inversion result of part b).

e) Estimate the effective depth of the inversion layer in strong inversion from ε_{si} and the value of C_{inv}. From the charge-screening intuition of Figure 3.8, explain why the depth of the inversion layer decreases as the inversion charge increases.

f) An electrochemical capacitance is formed by a metal electrode immersed in an ionic solution: The charged electrode creates field lines that terminate on ionic charges within a 'Debye layer' in the solution. The Debye layer is a layer analogous to the inversion layer, but with both negative and positive mobile charge. Provide an intuitive explanation for the cosh term of Equation (25.22), which describes how this electrochemical capacitance varies with V_{elec}, analogous to the surface potential, ψ_S. Can you predict the 'Debye-layer depth' from your knowledge of MOS device physics and inversion-layer depth?